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CONTACT PROBLEM OF ELASTICITY THEORY FOR A HALF-STRIP

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The problem of the effect of a die on an elastic semi-infinite strip fixed rigidly along the short edge is considered. Integral equations for the contact pressure and normal stress at the clamping are formed. These equations are reduced to two systems of linear algebraic equations by the Bubnov-Galerkin method. Both systems turn out to be well specified, and their coefficient matrices are almost triangular.

Numerical computations were carried out for a die with a flat bottom, for an oblique and a parabolic die, and the high efficiency of the method was shown.

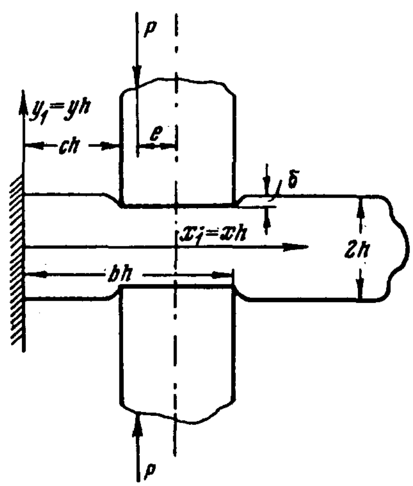

Fig. 1

1. Let us consider the problem of compressing a half-strip by two symmetrically disposed rigid dies under the following boundary conditions (Fig. 1):

$$u=v=0, \quad x=x_1/h=0$$
$$|y|=h^{-1}|y_1|\leqslant 1 \qquad (1.1)$$
$$\tau_{x_1y_1}=0, \qquad y=\pm 1, \qquad 0\leqslant x<\infty \qquad (1.2)$$
$$\sigma_{y_1}=0, \qquad y=\pm 1, \qquad 0\leqslant x\leqslant c \qquad x\geqslant b \qquad (1.3)$$

$$-v=[\delta-f^{*}(x)]\operatorname{sgn} y, \qquad y=\pm 1 \qquad c\leqslant x\leqslant b \tag{1.4}$$

Here u, v are displacements along the x_1 and y_1 axes, respectively, and $\tau_{x_1y_1}$, σ_{y_1} are the tangential and normal stresses; $f^*(x)$ is the equation of the die surface; δ the total displacement of the die under the effect of a force P. In [1] the problem for a half-strip with the boundary conditions (1.1), (1.3) is reduced to a Fredholm integral equation of the first kind for the normal stress at the clamping $\sigma(\eta)$. After simple manipulations this equation becomes

$$\int_{-1}^{1}\sigma(\eta)\,\Phi(\eta, y)\,d\eta+\int_{c}^{b}\sigma_{y_1}(r)\,\omega^{*}(r, y)\,dr+C_3=0 \tag{1.5}$$

$$\Phi(\eta, y)=\frac{\Phi^{*}(\eta, y)+\Phi^{*}(-\eta, y)}{2}, \qquad \Phi^{*}(\eta, y)=\ln|\eta-y|+$$

$$+\frac{\nu^2+2\nu+2}{\nu(\nu+2)}\ln[(2-\eta)^2-y^2]-\frac{2\nu}{(\nu+2)}(1-\eta)\left[\frac{(1-y)}{(2-y-\eta)^2}+\frac{(1+y)}{(2+y-\eta)^2}\right]+$$

$$+\ln[(4-\eta)^2-y^2]-\frac{4\nu}{(\nu+2)}\left[\frac{1}{4-y-\eta}+\frac{1}{4+y-\eta}\right]-$$

$$-\frac{\nu}{4(\nu+2)}\sum_{s=0}^{\infty}L_s(y)\,\eta^{2s+2}-\Pi(y), \qquad \nu=\frac{1}{1-2\mu}$$

$$L_s(y)=\sum_{k=0}^{\infty}y^{2k+2}\left(\frac{1}{4}\right)^{k+s}\frac{\Gamma(2k+2s+4)}{\Gamma(2k+3)\,\Gamma(2s+3)}\left[-\left(k+s+2.5+\frac{1}{\nu}\right)I_{2k+2s+3}+\right.$$

$$\left.+\left(2s+2+\frac{1}{\nu}\right)\left(2k+2+\frac{1}{\nu}\right)Q_{2k+2s+3}+(k+s+2)(k+s+2.5)\,Q_{2k+2s+5}\right]$$

$$Q_n=\frac{1}{2\Gamma(n+1)}\int_{0}^{\infty}\frac{e^{-\lambda}(2\lambda-e^{-\lambda})\,\lambda^{n}d\lambda}{(\operatorname{sh}\lambda+\lambda)}, \qquad \Gamma(n+1)=n\Gamma(n) \tag{1.6}$$

$$I_n=\frac{1}{2\Gamma(n+1)}\int_{0}^{\infty}\frac{e^{-\lambda}(2\lambda-e^{-\lambda})(\lambda-1-e^{-\lambda})\,\lambda^{n}d\lambda}{(\operatorname{sh}\lambda+\lambda)} \tag{1.7}$$

$$\Pi(y)=\frac{2}{(\nu+2)}\sum_{k=0}^{\infty}\left(\frac{y^2}{4}\right)^{k+1}\left\{-\left(\nu+\frac{2+\nu}{2+2k}\right)I_{2k+1}+\right.$$

$$\left.+\left[2+\frac{1}{\nu(k+1)}\right]Q_{2k+1}+\nu(k+1.5)\,Q_{2k+3}\right\}$$

$$\omega^{*}(r, y)=\int_{0}^{\infty}\frac{\sin\lambda r}{\Delta_{+}}\left[(\operatorname{ch}\lambda\operatorname{ch}\lambda y-y\operatorname{sh}\lambda\operatorname{sh}\lambda y)-\frac{1}{\nu}\operatorname{ch}\lambda y\frac{\operatorname{sh}\lambda}{\lambda}\right]d\lambda \tag{1.8}$$

$$\Delta_{+}=\operatorname{sh}2\lambda+2\lambda \tag{1.9}$$

Here C_3 is an arbitrary constant, μ the Poisson coefficient. Let us use the formulas obtained in [1] for the displacement v.

Then the contact condition (1.4) is written as follows:

$$\int_c^b \sigma_{y_1}(z) N(z, x)\, dz - \frac{4\nu}{(1+\nu)} \int_{-1}^{1} \sigma(\alpha)\, \omega^*(x, \alpha)\, d\alpha = -[\delta - f^*(x)] \frac{E}{2(1-\mu^2)} \frac{\pi}{h},$$

$$c \leqslant x \leqslant b \tag{1.10}$$

$$N(z, x) = K(z-x) - K(z+x), \qquad K(k) = \int_0^\infty \frac{(\operatorname{ch} 2\lambda - 1)\cos \lambda k\, d\lambda}{\lambda \Delta_+} \tag{1.11}$$

Here $\omega^*(x, \alpha)$, Δ_+ are functions given by (1.8) and (1.9). Putting

$$x = c_0 + a\zeta,\quad r = c_0 + ap,\quad z = c_0 + at,\quad c_0 = {}^1/_2\,(b + c),\quad a = {}^1/_2\,(b - c)$$

$$f^*(x) = f(\zeta),\quad \omega^*(r, y) = \omega(p, y),\quad \sigma y_1(r) = -Q(p),\quad N^*(z, x) = N(t, \zeta)$$

we rewrite the relationships (1.5),(1.10) as follows:

$$\int_{-1}^{1} \sigma(\eta)\, \Phi(\eta, y)\, d\eta - a \int_{-1}^{1} Q(p)\, \omega(p, y)\, dp + C_3 = 0, \qquad |y| \leqslant 1 \tag{1.12}$$

$$\int_{-1}^{1} Q(t)\, N(t, \zeta)\, dt + \frac{4\nu}{(1+\nu)} \frac{1}{a} \int_{-1}^{1} \sigma(\alpha)\, \omega(\zeta, \alpha)\, d\alpha - \frac{\pi}{ha}[\delta - f(\zeta)]\, \Delta_0 = 0$$

$$\Delta_0 = \frac{E}{2(1-\mu^2)}, \qquad |\zeta| \leqslant 1 \tag{1.13}$$

The problem is therefore reduced to solving a system of integral equations (1.12) – (1.13) in two unknowns: the normal stress at the clamping $\sigma(\eta)$, and the contact pressure under the die $Q(t)$.

It should be noted that the considered problem is a two-parameter problem with the dimensionless parameters $c_0 = {}^1/_2\,(c + b)$, $a = {}^1/_2(b - c)$ (Fig. 1).

Let us turn to the determination of the contact pressure $Q(t)$.

2. Let us first put $Q(p) = \delta(p - t)$ in (1.12), where $\delta(p - t)$ is the Dirac delta function. We determine $\sigma_0(\eta, t)$ from the obtained relationship.

Having studied the nature of the stress singularity at the clamping in [1], we chose the Bubnov-Galerkin method as the numerical method of solving (1.12). We assume

$$\sigma_0(\eta, t) = a\left[(1-\eta^2)^{p_0-1} E_0(t) + \sum_{n=0}^{n=m} F_n(t)\, T_{2n}(\eta)\right] \tag{2.1}$$

Here $T_{2n}(\eta)$ is an even Chebyshev polynomial of the first kind; $E_0(t)$, $F_n(t)$ are unknown coefficients; p_0 is the least positive root of the characteristic equation

$$2\varkappa \cos \pi p_0 - 4 p_0^2 + 1 + \varkappa^2 = 0, \qquad \varkappa = 3 - 4\mu \tag{2.2}$$

Let us substitute (2.1) into the integral equation (1.12), and let us demand that the expression obtained be orthogonal to $(1 - y^2)^{-1/2} T_{2k}(y)$ on the segment $[-1, 1]$. We hence obtain a system of linear algebraic equations in $E_0(t)$, $F_n(t)$, $C_3(t)$. The arbitrary constant $C_3(t)$ is determined from the static equilibrium condition

$$\int_{-1}^{1} \sigma_0(\eta, t)\, d\eta = 0 \tag{2.3}$$

Starting from the asymptotic expansion of the integral $\omega(t,y)$ in residues of the integrand, we represent $E_0(t)$, $F_n(t)$, $C_3(t)$ as

$$E_0(t)=\frac{2(1+\nu)}{(\nu+2)}\sum_{k=0}^{k=l} e^{-\beta_k\gamma}(\cos\alpha_k\gamma B_{k1}-\sin\alpha_k\gamma B_{k2}) \tag{2.4}$$

$$F_n(t)=\frac{2(1+\nu)}{(\nu+2)}\sum_{k=0}^{k=l} e^{-\beta_k\gamma}[\cos\alpha_k\gamma A_{n1}(k)-\sin\alpha_k\gamma A_{n2}(k)] \tag{2.5}$$

$$C_3(t)=\frac{2(1+\nu)}{(\nu+2)}\sum_{k=0}^{k=l} e^{-\beta_k\gamma}[\cos\alpha_k\gamma C_{l,1}-\sin\alpha_k\gamma C_{l,2}]$$

$$\alpha_k+i\beta_k=\lambda_k,\ \operatorname{sh}\lambda_k+\lambda_k=0,\ \alpha_k>0,\ \beta_k>0,\ \gamma={}^1/_2(c_0+at)\geqslant\gamma_0\neq 0 \tag{2.6}$$

Here γ_0 is the lower bound of the variable γ. The system of linear algebraic equations in the unknowns $E_0(t)$, $F_n(t)$, $C_3(t)$ then goes over into a system of linear algebraic equations in the new unknowns B_{k1}, $A_{n1}(k)$, C_{k1} and B_{k2}, $A_{n2}(k)$, C_{k2}. For any arbitrary contact pressure $Q(t)$ the stress at the clamping is determined from the relationship

$$\sigma(\eta)=\int_{-1}^{1} Q(t)\,\sigma_0(\eta,t)\,dt \tag{2.7}$$

where $\sigma_0(\eta, t)$ is given by (2.1).

3. Let us substitute (2.7) into (1.11). We hence obtain an integral equation in the contact pressure

$$\int_{-1}^{1} Q(t)\,\Omega(t,\zeta)\,dt-[\delta-f(\zeta)]\,\Delta_0\frac{\pi}{ha}=0 \tag{3.1}$$

$$\Omega(t,\zeta)=K[a(\zeta-t)]-K[2c_0+a(\zeta+t)]+\frac{4\nu}{(1+\nu)a}\psi(t,\zeta)$$

$$\psi(t,\zeta)=\int_{-1}^{1}\sigma_0(\alpha,t)\,\omega(\zeta,\alpha)\,d\alpha \tag{3.2}$$

The following properties of the kernel have been established in [2]:

a) For k $(-\infty, \infty)$

$$K(k)=-\ln k+F(k) \tag{3.3}$$

where $F(k)$ is an even continuous function in all its derivatives with respect to k.

b) For $k\to\infty$ the kernel $K(k)$ tends exponentially to zero.

In the case ${}^1/_2(c_0+at)\geqslant\gamma_0$, ${}^1/_2(c_0+a\zeta)\geqslant\gamma_0$, $\gamma_0\neq 0$ the kernel $K[2c_0+a(\zeta+t)]$ is evidently a continuous function with all its derivatives with respect to ζ and t.

Let us investigate the properties of the function $\psi(t, \zeta)$. From (1.12) we have

$$a\omega(\zeta,\alpha)=C_3(\zeta)+\int_{-1}^{1}\sigma_0(\eta,\zeta)\,\Phi(\eta,\alpha)\,d\eta \tag{3.4}$$

Substituting (3.4) into (3.2) under the conditions (2.3), we obtain

$$\psi(t,\zeta)=\frac{1}{a^2}\int_{-1}^{1}d\alpha\int_{-1}^{1}\sigma_0(\alpha,t)\,\sigma_0(\eta,\zeta)\,\Phi(\eta,\alpha)\,d\eta \tag{3.5}$$

From (3.5) it follows:

a) The function $\psi(t,\zeta)=\psi(\zeta,t)$ because $\Phi(\eta,\alpha)$ is a symmetric kernel.

b) The function $\psi(t,\zeta)$ is continuously differentiable any number of times with respect to t and ζ, since $\sigma_0(\alpha, t)$ and $\sigma_0(\eta,\zeta)$ are continuous functions with all their derivatives with respect to t and ζ, respectively, in the domain ${}^1/_2(c_0+at)\geqslant\gamma_0$,

$^1/_2\,(c_0 + a\zeta) \geqslant \gamma_0,\ \gamma_0 \neq 0$.

c) For $^1/_2\,(c_0 + at) \to \infty,\ ^1/_2\,(c_0 + a\zeta) \geqslant \gamma_0$ the function $\psi(t, \zeta)$ decreases exponentially.

Since the free term in (3.1) is assumed continuous and continuously differentiable any number of times, the singularity in the solution of this equation will hence be defined by the logarithmic kernel contained in $K[a(\zeta - t)]$. then follows from [3] that the solution of (3.1) can be represented as

$$Q(t) = \frac{1}{\sqrt{1-t^2}} \sum_{k=0}^{k=N} G_k T_k(t) \tag{3.6}$$

where $T_k(t)$ is a Chebyshev polynomial of the first kind.

Proceeding from the representation (3.6), we reduce the integral equation (3.1) to a system of linear algebraic equations in G_k by the Bubnov-Galerkin method, just as this was done in [4]. This system can be written thus:

$$G_0\left(R_{00} + \ln\frac{2}{a} - C_{00}\right) + \sum_{s=1}^{s=N} \frac{(R_{0s} - C_{0s})}{2} G_s = \frac{\Delta_0}{\pi} p_0 \tag{3.7}$$

$$G_0(R_{m0} - C_{m0}) + \sum_{s=1}^{s=N} \frac{(R_{ms} - C_{ms})}{2} G_s + \frac{G_m}{m} = \frac{\Delta_0 p_m}{\pi}, \qquad 1 \leqslant m \leqslant N \tag{3.8}$$

where R_{ms}, C_{ms} are coefficients of the following expansions:

$$F\,[a(\zeta - t)] = K\,[a(\zeta - t)] + \ln a(\zeta - t) = \sum_{\substack{m=0\\s=0}}^{\infty} R_{ms} T_m(\zeta) T_s(t) \tag{3.9}$$

$$C_{ms} = \delta_{ms} + a_{ms}, \qquad K\,[2C_0 + a(\zeta + t)] = \sum_{\substack{m=0\\s=0}}^{\infty} a_{ms} T_m(\zeta) T_s(t) \tag{3.10}$$

$$\psi(t, \zeta) = \sum_{\substack{m=0\\s=0}}^{\infty} \delta_{ms} T_m(\zeta) T_s(t) \tag{3.11}$$

$$\frac{\pi}{ha}\,[\delta - f(\zeta)]\,\Delta_0 = \sum_{m=0}^{m=M} p_m \Delta_0 T_m(\zeta) \tag{3.12}$$

4. Computations were carried out for $\mu = 0.317408$, $p_0 = 0.700000$. Their representations in terms of the Howland integral [5] were used to evaluate Q_n, I_n given by (1.6), (1.7). The expanded matrix of the system in the unknowns $E_0(t)$, $F_n(t)$, $C_3(t)$ was computed by the mechanical quadrature method [6].

Values of the stresses in the framing were computed by means of (2.1) practically exactly for $m = 5$. Thus, in the case $\gamma = 0.25$ and $l = 1$, the fifth approximation ($m = 5$) in (2.4) – (2.6) differs by no more than 0.5% from the sixth ($m = 6$). However, the error with which the coefficients $E_0(t)$, $F_n(t)$ have been determined by means of (2.4), (2.5), and which arises because of discarded terms ($l \geqslant 2$), introduces an additional error in the found stresses which does not exceed 3.5%. Therefore, the total error in the solution $\sigma_0(\eta, t)$ for $\gamma = 0.25$ and $l = 1$ does nor exceed 4%.

This case corresponds to a quite close disposition of the die to the clamping.

In the case $\gamma \geqslant 0.5$, $l = 1$ the total error in the solution is less than 1%.

To calculate the coefficients B_{ms} in the expansion in (3.9) we used the representation

$$F\,[a\,(\zeta - t)] = \sum_{k=0}^{k=7} d_{2k}\,(a)\,T_{2k}\left(\frac{\zeta - t}{2}\right) \tag{4.1}$$

$$d_0\,(a) = -\,0.3516754 + 2\left\{\ln\,(1 + r) + \frac{a^2}{\sqrt{1 + a^2}\,(1 + \sqrt{1 + a^2})} - \right.$$

$$\left. - \frac{a^2}{\sqrt{4 + a^2}}\left[\frac{1}{(4 + a^2)} + \frac{1.5}{(2 + \sqrt{4 + a^2})}\right]\right\} + \sum_{p=0}^{\infty} B_{p+1}\varphi_p \tag{4.2}$$

$$r = \frac{3a^2}{(2 + \sqrt{4 + a^2})\,(2\sqrt{1 + a^2} + \sqrt{4 + a^2})}\,, \qquad \varphi_0 = a^2$$

$$\varphi_p = \varphi_{p-1}\,\frac{(-\,a^2)\,(p + 0.5)}{(p + 1)}\,, \qquad p \geqslant 1$$

$$d_{2m}\,(a) = 4\,(-1)^{m+1}\left\{\Gamma_m\,(1)\left(\frac{1}{2m} + \frac{1}{\sqrt{1 + a^2}}\right) - \right.$$

$$\left. -\,\Gamma_m\,(2)\left[\frac{1}{2m} + \frac{2}{\sqrt{4 + a^2}}\left(1 + \frac{2m}{\sqrt{4 + a^2}} + \frac{2}{4 + a^2}\right)\right] + \sum_{p=0}^{\infty} B_{p+m}\psi_p\right\} \tag{4.3}$$

$$m \geqslant 1,\ \Gamma_m\,(k) = \left[\frac{a}{k + \sqrt{k^2 + a^2}}\right]^{2m},\ \psi_0 = \left(\frac{a}{2}\right)^{2m},\ \psi_p = \psi_{p-1}\,\frac{(-a^2)\,(m + p)\,(m + p - 0.5)}{p\,(p + 2m)}$$

The values of B_p are represented in Table 1.

Table 1.

p	B_p	m	$d_{2m}(a)$		
			$a = 0.5$	$a = 1$	$a = 1.5$
1	0.2920749	0	−0.1328900	0.2512295	0.5711618
2	0.6397242 (−1)	1	0.2061846	0.4977991	0.6636256
3	0.1270032 (−1)	2	−0.1187582 (−1)	−0.8623488 (−1)	−0.1811443
4	0.2272334 (−2)	3	0.6844342 (−3)	0.1557360 (−1)	0.5494941 (−1)
5	0.374256 (−3)	4	−0.3833251 (−4)	−0.2731640 (−2)	−0.1641086 (−1)
6	0.57821 (−4)	5	0.2119323 (−5)	0.4694238 (−3)	0.4785882 (−2)
7	0.85047 (−5)	6	−0.1168139 (−6)	−0.7995242 (−4)	−0.1374129 (−2)
8	0.12040 (−5)	7	0.6443156 (−8)	0.1358443 (−4)	0.391576 (−3)
9	0.1656 (−6)	8		−0.2308797 (−5)	−0.111254 (−3)
10	0.222 (−7)	9		0.39286 (−6)	0.315990 (−4)
11	0.29 (−8)	10		−0.6693 (−7)	−0.897 (−5)
12	0.34 (−9)	11		0.1141 (−7)	0.255 (−5)
13	0.44 (−10)				

Note. −0.1187582 (−1) means −0.01187582.

The representation of $F[a(\zeta - t)]$ in terms of the Howland integral [5] was used in deriving (4.2), (4.3). Values of $d_{2m}(a)$ for a=0.5, 1.0, 1.5 are arranged in Table 1.

The expansions

$$T_2\,(^1/_2\,[\zeta - t]) = \sum_{\substack{s=0\\m=0}}^{s+m\leqslant 2k} b_{sm}\,(k)\,T_s\,(t)\,T_m\,(\zeta) \tag{4.4}$$

for arbitrary k were obtained by successive application of the relationship

$$T_{k+1}(^1/_2[\zeta - t]) = -\,T_{k-1}(^1/_2[\zeta - t]) + T_k(^1/_2[\zeta - t])[T_1(\zeta) - T_1(t)] \tag{4.5}$$

Substituting (4.4) into (4.1), and collecting like terms, we obtain the values of the coefficients R_{ms} on the basis of the expansion (3.9). Representations of (3.10), (3.11) in terms of Bessel functions of complex argument were used in determining the coefficients C_{ms}. Hence, when three terms ($M = 2$) are retained in the expansion (3.12), the solution of (3.1) is

$$Q(t) = \frac{\Delta_0}{\sqrt{1-t^2}} \sum_{m=0}^{m=2} p_m g_m, \qquad g_m = \sum_{k=0}^{k=N} G_{mk} T_k(t) \tag{4.6}$$

Values of the coefficients G_{mk} in the expression in (4.6) are presented in Table 2 for $a = 0.5$, $c_0 = 1$, $N = 4$ and $a = 1$, $c_0 = 1.5$, $N = 6$.

Table 2.

a	m	G_{m0}	G_{m1}	G_{m2}	G_{m3}	G_{m4}	G_{m5}	G_{m6}
0.5	0	0.29454	—0.01713	—0.02699	0.00094	0.00023		
	1	—0.00856	0.36922	—0.00422	—0.00251	0.00002		
	2	—0.01350	—0.00423	0.64774	—0.00124	—0.00024		
1	0	0.49539	—0.02532	—0.11647	0.00035	0.00405	0.00093	—0.00031
	1	—0.01266	0.47230	—0.00886	—0.02916	0.00011	0.00206	0.00020
	2	—0.05822	—0.00886	0.71427	—0.00456	—0.01000	—0.00037	0.00102

The found approximations $N = 4$ ($a = 0.5$) and $N = 6$ ($a = 1$) are practically exact since they differ from the next approximations $N = 6$ ($a = 0.5$) and $N = 8$ ($a = 1$) by not more than 0.02%.

If it is here taken into account that the normal stresses at the clamping have been computed with not more than 4% error, then the total error in the solutions found $N = 4$ ($a = 0.5$) and $N = 6$ ($a = 1$) does not exceed 1%.

By selecting appropriate values of p_0, p_1, p_2 contact pressures for three kinds of dies can be computed: for a die with a flat base, for an oblique die, and for a parabolic die.

Presented in Table 3 are values of the contact pressures computed by means of (4.6) for the cases $N = 4$ ($a = 0.5$) and $N = 6$ ($a = 1$) when $p_0 = \pi\delta/ha$, $p_1 = p_2 = 0$ (die with a flat base).

From an analysis of Table 3 it follows that the contact pressures rise as the point under consideration approaches the clamping, whereupon a tilting moment acting on the die is manifest.

Therefore, the force P must be applied at a distance $e/h = {}^1/_2\, a\, G_{01}/G_{00}$ (Fig. 1) from the axis of symmetry of the die (Fig. 1).

Values of $Q(t)$ obtained herein for $N = 6$, $a = 1$, $c_0 = 1.5$, differ by not more than 14% from the values of the contact pressures computed in [4] in the absence of clamping.

Values of the stresses at the clamping computed

Table 3.

t	$Q(t)\dfrac{h}{\pi\delta}\dfrac{1}{\Delta_0}$, $a = 0.5$	$Q(t)\dfrac{h}{\pi\delta}\dfrac{1}{\Delta_0}$, $a = 1$
—0.95105	1.8673	1.3798
—0.90631	1.3839	1.0494
—0.80901	1.0213	0.81303
—0.58779	0.77542	0.67127
—0.42261	0.70616	0.63948
—0.17365	0.65704	0.62140
0.00000	0.64354	0.61623
0.17365	0.64306	0.61355
0.42261	0.67022	0.61679
0.58779	0.72122	0.63364
0.80901	0.92506	0.73980
0.90631	1.2392	0.93867
0.95105	1.6635	1.2253

Table 4

t	$\sigma(t)\,h\,/\,C$, $a = 0.5$	$\sigma(t)\,h\,/\,C$, $a = 1$
0.02079	—0.14009	—0.09276
0.12050	—0.14088	—0.09263
0.21901	—0.14231	—0.09208
0.40849	—0.13876	—0.08545
0.58168	—0.09590	—0.05466
0.79608	0.11657	0.07768
0.95534	0.52336	0.31954

by (2.1) and (2.7), are presented in Table 4. The coefficients $E_0(t)$, $F_n(t)$ were hence represented as

$$E_0(t) = \sum_{s=0}^{s=9} l_s T_s(t), \qquad F_n(t) = \sum_{s=0}^{s=9} e_s(n) T_s(t)$$

Representations of (2.4), (2.5) in terms of Bessel functions of complex argument were used to determine the coefficients l_s and $e_s(n)$.

It is seen from Table 4 that the stresses in the framing diminish as the the zone of contact increases. All the computations were made on the "Minsk-12" computer.

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